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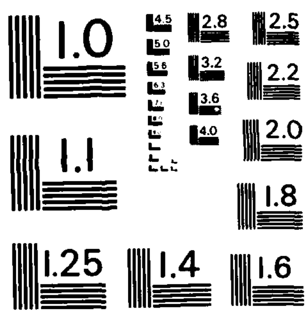

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NRL Report 8731

Multisignal Minimum-Cross-Entropy Spectrum Analysis with Weighted Priors

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This report presents a generalization of multisignal minimum-cross-entropy spectrum analysis (multisignal MCESA), a method for simultaneously estimating a number of power spectra when a prior estimate of each is available and new information is obtained in the form of values of the autocorrelation function of their sum. The generalization involves attaching to each prior spectrum estimate a frequency-dependent weighting parameter that indicates its relative reliability, or the relative "degree of belief" associated with it. Mathematical properties of the generalized method are discussed, and illustrative numerical examples are given.

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CONTENTS

MULTISIGNAL MINIMUM-CROSS-ENTROPY SPECTRUM ANALYSIS WITH WEIGHTED PRIORS

INTRODUCTION

Multisignal minimum-cross-entropy spectrum analysis (MCESA) is a method for estimating the power spectrum of one or more signals when a *prior* estimate for each is available and new information is obtained in the form of values of the autocorrelation function of their sum [1]. The resultant estimates are the solution of a constrained minimization problem: they are consistent with the autocorrelation information and otherwise as similar as possible to the respective prior estimates in a precisely defined information-theoretic sense. Multisignal MCESA [1] is a generalization of MCESA [1,2] and reduces to it in the special case when the number of signals is one. Multisignal MCESA can be derived [1,2] as an application of the principle of minimum cross entropy [3-5] or, alternatively, by minimizing a sum of Itakura-Saito distortions [6].

Multisignal MCESA applies when, for instance, one obtains autocorrelation measurements for a signal corrupted by independent additive noise, and one has some prior knowledge concerning the spectra of both the uncorrupted signal and the noise. The results are posterior signal- and noise-spectrum estimates that take both the prior estimates and the autocorrelation information into account.

The multisignal MCESA procedure presented in Ref. 1 gives the same weight to each of the prior spectrum estimates—they are all treated on the same footing. However, in situations that arise in practice, one may have more reliable information about the spectra of some of the signals than about the others. Consider a speech signal corrupted by additive background noise; suppose the background is more nearly stationary than speech. If it is possible to detect pauses in the speech reliably, then measurements of the sum signal during a speech-free interval will yield an estimate of the noise spectrum that can serve as a prior noise estimate during an interval when speech is present. The result may well be a better prior estimate for the noise spectrum than any prior estimate that can be obtained for the speech spectrum. Alternatively one might be able to obtain a good estimate of the noise power spectrum by conventional spectrum analysis of a signal from a microphone exposed to the noise but not the speech. In both cases it would be desirable to give greater weight to the noise prior than to the speech prior in deriving posterior spectrum estimates. In other situations it might be desirable to rely more heavily on the speech prior than on the noise prior.

It is furthermore possible to have prior information about the spectrum of an individual signal that is more reliable in some frequency ranges than in others. Thus in some situations, for example, one might even wish to give greater weight to the noise prior at high frequencies and to the speech prior at low frequencies.

We present in this report a generalization of multisignal MCESA that allows a frequency-dependent weight to be attached to each prior estimate. Aside from the weights, inputs to the procedure are the same as to multisignal MCESA: a prior spectrum estimate for each signal and autocorrelation values for the sum. The results again are posterior spectrum estimates that are consistent with

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the given autocorrelation information. When all of the weights are constant and equal, the results are identical to those from multisignal MCESA. In general, increasing a weighting parameter tends to bring the corresponding posterior spectrum closer (in the sense of Itakura-Saito distortion) to the corresponding prior spectrum at the expense of increasing the distortions for other prior-posterior pairs. This is demonstrated in the fourth section of this report, where it is also shown that the posterior spectra can be obtained by constrained minimization of a weighted sum of Itakura-Saito distortions. In the remainder of this section, we describe the multisignal MCESA method of [1]. In the second section of this report, we present the generalized method and the heuristic arguments that first led us to the generalization. In the third section we give a derivation based on minimizing a weighted sum of cross entropies, and in the fifth section we rework a numerical example from Ref. 1.

Multisignal MCESA estimates the power spectra $S_i(f)$ of a number K of independent, real, bandlimited, stationary processes (signals) with bandwidth W, given a prior estimate P_i for each S_i, and given in addition the values $R_r^{\text{tot}} = R^{\text{tot}}(t_r)$ of the autocorrelation function of the sum of the processes at finitely many lags t_r, $r = 0, \ldots, M$. The prior estimates P_i may be thought of as the best guesses at S_i we could make in the absence of the autocorrelation data. Under the assumption of independence, we write R^{tot} as a sum of autocorrelation functions R_i of individual processes:

$$R^{\text{tot}}(t) = \sum_{i=1}^{K} R_i(t),$$

where

$$R_i(t) = 2\int_0^W S_i(f) \cos 2\pi ft \, df.$$

The multisignal MCESA estimator has the form [1]

$$Q_i(f) = \frac{1}{\dfrac{1}{P_i(f)} + \sum_{r=1}^{M} 2\beta_r \cos 2\pi f t_r}, \tag{1}$$

where the β_r are chosen so that the Q_i are consistent with the given autocorrelation values:

$$R_r^{\text{tot}} = 2\sum_{i=1}^{K} \int_0^W Q_i(f) \cos 2\pi f t_r \, df. \tag{2}$$

The parameters β_r are Lagrange multipliers that arise in the solution of a minimization problem with Eq. (2) as constraints. The minimization problem can be formulated as an application of the principle of minimum cross entropy or, alternatively, as the minimization of the sum

$$\sum_{i=1}^{K} \int_0^W \left[\frac{Q_i(f)}{P_i(f)} - \log \frac{Q_i(f)}{P_i(f)} - 1 \right] df \tag{3}$$

of Itakura-Saito distortions.

THE METHOD

To use the multisignal MCESA estimate with weighted priors, we must supply not only autocorrelation values R_r^{tot} for the sum of the signals and prior estimates P_i for the individual signals, but also a weight w_i associated with each P_i. The w_i may be simply K positive constants or, more generally, K functions $w_i(f)$ of frequency. The estimate has the form

$$Q_i(f) = \frac{1}{\dfrac{1}{P_i(f)} + \dfrac{1}{w_i(f)} \sum_{r=1}^{M} 2\beta_r \cos 2\pi f t_r}, \tag{4}$$

where the parameters β_r are to be chosen so that the constraints of Eq. (2) are satisfied.

We first arrived at something like Eq. (4), with frequency-independent weights, by considering a somewhat artificial situation. Suppose that each signal s_i is the sum of n_i independent signals s_{ij} with spectra S_{ij}, and that for each i we have equal prior estimates P_i/n_i for all the S_{ij}, $j = 1, \ldots, n_i$. Then we might consider n_i to be a measure of the *quality* or *reliability* of P_i as an estimate of $S_i = \sum_j S_{ij}$, by analogy with the fact that the sum of n independent random variables distributed identically with x/n has a smaller variance (by a factor of n) than a single random variable x. The MCESA posterior estimates Q_{ij} for S_{ij} in this situation are given by

$$Q_{ij}(f) = \frac{1}{\dfrac{1}{P_i(f)/n_i} + \sum_{r=1}^{M} 2\beta_r \cos 2\pi f t_r},$$

where the β_r are chosen to satisfy constraints

$$R_r^{\text{tot}} = 2\sum_{i=1}^{K} \sum_{j=1}^{n_i} \int_0^W Q_{ij}(f) \cos 2\pi f t_r \, df. \tag{5}$$

We consequently obtain

$$\begin{aligned} Q_i(f) &= \sum_{j=1}^{n_i} Q_{ij}(f) = n_i Q_{ij}(f) \\ &= \frac{1}{\dfrac{1}{P_i(f)} + \dfrac{1}{n_i}\sum_{r=1}^{M} 2\beta_r \cos 2\pi f t_r}, \end{aligned} \tag{6}$$

as a reasonable posterior estimate of $S_i(f)$. When Q_i is given by Eq. (6), the constraints of Eq. (5) become equivalent to those of Eq. (2).

The posterior spectra Q_i are not altered if the n_i are replaced with a proportional family of numbers w_i, since a common factor can be absorbed into the β_r. Furthermore, there does not seem to be much point to requiring the ratios of the w_i to be rational—we regard the model of s_i as a sum of components s_{ij} as being suggestive, but not necessarily to be taken literally. We therefore replace n_i in Eq. (6) with arbitrary positive numbers w_i and obtain

$$Q_i(f) = \frac{1}{\dfrac{1}{P_i(f)} + \dfrac{1}{w_i}\sum_{r=1}^{M} 2\beta_r \cos 2\pi f t_r}. \tag{7}$$

Allowing the weights to be frequency-dependent can be motivated heuristically by considering another somewhat artificial procedure—converting a single-signal problem into a multisignal problem. Assume we are given autocorrelation values R_r and a prior estimate P for the spectrum S of a single signal s. Suppose we arbitrarily partition the band of frequencies from 0 to W into two bands B_1 and B_2 and write s as a sum

$$s(t) = s_1(t) + s_2(t)$$

of two signals whose spectra S_1 and S_2 are confined to the respective bands B_1 and B_2

$$S_i(f) = \begin{cases} S(f), & f \in B_i \\ 0, & f \notin B_i \end{cases},$$

$r = 1, 2$. We solve the two-signal MCESA problem with autocorrelation values given by $R_r tot = R_r$, prior estimates P_1, P_2 given by

$$P_i(f) = \begin{cases} P(f), & f \in B_i \\ 0, & f \notin B_i \end{cases},$$

and frequency-independent weights w_i. Strictly speaking, Eq. (4) is ill-defined where $P_i(f) = 0$; however, there is no problem with the following alternative form,

$$Q_i(f) = \frac{P_i(f)}{1 + \frac{1}{w_i} P_i(f) \sum_{r=1}^{M} 2\beta_r \cos 2\pi f t_r}.$$

The sum Q of the posteriors satisfies

$$\begin{aligned} Q(f) &= Q_1(f) + Q_2(f) \\ &= \begin{cases} Q_1(f), & f \in B_1 \\ Q_2(f), & f \in B_2 \end{cases}, \end{aligned}$$

and therefore

$$Q(f) = \frac{1}{\frac{1}{P(f)} + \frac{1}{w_i} \sum_{r=1}^{M} 2\beta_r \cos 2\pi f t_r},$$

when $f \in B_i$, $(i = 1, 2)$. With equal weights $w_1 = w_2 = 1$, we thus merely recover the single-signal MCESA result in a roundabout way. However, we may write this as

$$Q(f) = \frac{1}{\frac{1}{P(f)} + \frac{1}{w(f)} \sum_{r=1}^{M} 2\beta_r \cos 2\pi f t_r}, \tag{8}$$

where, with general weights,

$$w(f) = \begin{cases} w_1, & f \in B_1 \\ w_2, & f \in B_2 \end{cases}$$

We could equally well have partitioned the frequencies into any number of sets B_i, not merely 2; from that point it is an obvious step to consider Eq. (8) with weights $w(f)$ not restricted to a finite number of values w_i. We are thus led to a single-signal version of Eq. (4). The generalizations that lead from the unweighted multisignal MCESA estimate Eq. (1) to Eq. (7) and Eq. (8) combine to yield Eq. (4) in full generality.

DERIVATION

We collect here some notation and results from Ref. 1. For the K signals, we use discrete-spectrum approximations

$$s_i(t) = \sum_{k=1}^{N} (a_{ik} \cos 2\pi f_k t + b_{ik} \sin 2\pi f_k t),$$

$(i = 1, \ldots, K)$. The f_k are nonzero frequencies, not necessarily uniformly spaced, and the a_{ik} and b_{ik} are random variables. We define random variables

$$x_{ik} = \frac{1}{4}(a_{ik}^2 + b_{ik}^2)$$

representing the power of process s_i at frequency f_k, and we consider their joint probability density $q^\dagger(\mathbf{x})$, where $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_K)$ and $\mathbf{x}_i = (x_{i1}, \ldots, x_{iN})$. Assuming prior estimates $P_{ik} = P_i(f_k)$ of the power spectra of the s_i, we write prior estimates p of $q^\dagger$ in the form

$$p(\mathbf{x}) = \prod_{i=1}^{K} \prod_{k=1}^{N} p_{ik}(x_{ik}),$$

where

$$p_{ik}(x_{ik}) = \frac{1}{P_{ik}} \exp \frac{-x_{ik}}{P_{ik}}.$$

Autocorrelation values $R_r tot = R(t_r)$ for the sum of the s_i are assumed known at lags t_r $(r = 0, \ldots, M)$ with $t_0 = 0$. (In Ref. 1, R_r is written in place of $R_r tot$.) We write these as linear constraints

$$R_r^{\text{tot}} = \sum_{i=1}^{K} \sum_{k=1}^{N} \int c_{rk} x_{ik} q^{\dagger}(\mathbf{x})\, d\mathbf{x} \tag{9}$$

on expectation values of $q^{\dagger}$, where

$$c_{rk} = 2 \cos 2\pi f_k t_r.$$

We obtain a posterior estimate q of $q^{\dagger}$ by minimizing the cross entropy

$$H(q, p) = \int q(\mathbf{x}) \log \frac{q(\mathbf{x})}{p(\mathbf{x})}\, d\mathbf{x}$$

subject to the constraints (Eq. (9) with q in place of $q^{\dagger}$); the result has the form

$$q(\mathbf{x}) = \prod_{i=k}^{K} \prod_{k=1}^{N} q_{ik}(x_{ik}), \tag{10}$$

where the q_{ik} are related to the posterior estimates

$$Q_{ik} = Q_i(f_k) = \int x_{ik} q(\mathbf{x})\, d\mathbf{x}$$

of the power spectra of the s_i by

$$q_{ik}(x_{ik}) = \frac{1}{Q_{ik}} \exp \frac{-x_{ik}}{Q_{ik}}. \tag{11}$$

We find a discrete-frequency version of Eq. (1),

$$Q_{ik} = \frac{1}{\dfrac{1}{P_{ik}} + \sum_{r=1}^{M} \beta_r c_{rk}}, \tag{12}$$

where the β_r must be chosen so that

$$\sum_{i=1}^{K} \sum_{k=1}^{N} c_{rk} Q_{ik} = R_r^{\text{tot}}$$

is satisfied.

Equation (10) states the posterior independence of the x_{ik}. We would have obtained the same results Eq. (11), Eq. (12) if we had assumed Eq. (10) from the start, choosing the q_{ik} to minimize $H(q, p)$ subject to the constraints with q expressed in the form of Eq. (10). For such densities q, we have

$$H(q, p) = \sum_{i=1}^{K} \sum_{k=1}^{N} H(q_{ik}, p_{ik}), \tag{13}$$

and the constraints assume the form

$$\sum_{i=1}^{K} \sum_{k=1}^{N} \int c_{rk} x_{ik} q_{ik}(x_{ik})\, dx_{ik} = R_r^{\text{tot}}. \tag{14}$$

Thus multisignal MCESA (without weighting factors) can be obtained by minimizing Eq. (13) subject to the constraints of Eq. (14). Our generalization is to replace the right-hand side of Eq. (13) with a weighted sum

$$\sum_{i=1}^{K}\sum_{k=1}^{N} w_{ik}\, H(q_{ik}, p_{ik}) = \sum_{i=1}^{K}\sum_{k=1}^{N} w_{ik}\int q_{ik}(x_{ik}) \log \frac{q_{ik}(x_{ik})}{p_{ik}(x_{ik})}\, dx_{ik}.$$

This is to be minimized with respect to variations in the q_{ik} subject to the constraints Eq. (14)—together, of course, with the normalization constraints

$$\int q_{ik}(x_{ik})\, dx_{ik} = 1. \tag{15}$$

The result is

$$q_{ik}(x_{ik}) = p_{ik}(x_{ik}) \exp\left[-1 - \frac{\lambda_{ik}}{w_{ik}} - \frac{1}{w_{ik}}\sum_{r=1}^{N} \beta_r c_{rk} x_{ik}\right],$$

where the β_r and the λ_{ik} are Lagrange multipliers corresponding to the constraints Eqs. (14) and (15). From this it follows that Eq. (11) holds with

$$\begin{aligned} Q_{ik} &= \int x_{ik} q_{ik}(x_{ik})\, dx_{ik} \\ &= \frac{1}{\dfrac{1}{P_{ik}} + \dfrac{1}{w_{ik}}\displaystyle\sum_{r=1}^{M} \beta_r c_{rk}}; \end{aligned} \tag{16}$$

the argument is the same that established (13) and (14) from Eq. (11) in Ref. 2.

Passing to the continuous-frequency case, we write the weights as $w_i(f)$ and replace Eq. (16) with Eq. (4).

PROPERTIES

We begin with two trivial observations. The first is that, as noted in the second section of this report, the weights w_i may be scaled by a common factor without affecting the results. The second is that the present method does indeed reduce to multisignal MCESA when the w_i are all constant and equal.

Next we show that the posterior spectra Q_i can be obtained by minimizing the sum

$$\sum_{i=1}^{K}\int_0^W w_i(f)\left[\frac{Q_i(f)}{P_i(f)} - \log\frac{Q_i(f)}{P_i(f)} - 1\right] df \tag{17}$$

of weighted Itakura-Saito distortions subject to the constraints of Eq. (2). We form the expression

$$\sum_{i=1}^{K}\int_0^W w_i(f)\left[\frac{Q_i(f)}{P_i(f)} - \log\frac{Q_i(f)}{P_i(f)} - 1\right] df + \sum_{r=1}^{M} 2\beta_r \sum_{i=1}^{K}\int_0^W Q_i(f)\cos 2\pi f t_r\, df$$

involving Lagrange multipliers β_r, and we set its variation with respect to $Q_i(f)$ equal to zero:

$$w_i(f)\left[\frac{1}{P_i(f)} - \frac{1}{Q_i(f)}\right] + \sum_{r=1}^{M} 2\beta_r \cos 2\pi f t_r = 0.$$

This implies Eq. (4). We obtain a minimum of Eq. (17) since the second variation, $w_i(f)/Q_i(f)^2$, is positive.

Finally, we justify the claim that increasing one of the weights tends to decrease the distortion between the corresponding prior and posterior spectra while increasing the distortions for other prior-posterior pairs. We write $D(Q_i,P_i)$ for the Itakura-Saito distortion of P_i with respect to Q_i (the integral in Eq. (3)). We first consider frequency-independent weights. Let w'_i be the result of increasing w_i for a particular value a of i and leaving the rest of the weights the same:

$$w'_a > w_a,$$

$$w'_i = w_i \quad (i \neq a).$$

Let the use of weights w'_i result in posterior spectra Q'_i. We will show that

$$D(Q'_a,P_a) < D(Q_a,P_a), \tag{18}$$

and that

$$D(Q'_i,P_i) > D(Q_i,P_i) \tag{19}$$

for at least one value of i (necessarily different from a).

Now Q_i minimizes

$$\sum_i w_i D(Q_i,P_i)$$

subject to the constraints (2) while Q'_i minimizes

$$\sum_i w'_i D(Q'_i,P_i)$$

subject to the same constraints (with Q'_i in place of Q_i). It follows that

$$\sum_i w_i D(Q_i,P_i) < \sum_i w_i D(Q'_i,P_i) \tag{20}$$

and

$$\sum_i w'_i D(Q_i,P_i) > \sum_i w'_i D(Q'_i,P_i). \tag{21}$$

Subtracting Eq. (20) from Eq. (21) yields

$$(w'_a - w_a) D(Q_a,P_a) > (w'_a - w_a) D(Q'_a,P_a).$$

Since $(w'_a - w_a)$ is positive, we have Eq. (18). But it follows from Eq. (20) that Eq. (19) holds for some i.

A similar argument establishes a somewhat similar result for frequency-dependent weights. For simplicity we state this for the single-signal case. Let w' be the result of increasing w on some band B of frequencies:

$$w'(f) > w(f), \quad f \in B$$

$$w'(f) = w(f), \quad f \notin B.$$

Let the use of w and w' result in posterior spectra Q and Q', respectively. Then

$$\int_{B'} \left[\frac{Q'(f)}{P(f)} - \log \frac{Q'(f)}{P(f)} - 1 \right] df < \int_{B'} \left[\frac{Q(f)}{P(f)} - \log \frac{Q(f)}{P(f)} - 1 \right] df$$

for some $B' \subseteq B$, while the reverse inequality holds for some B'' disjoint from B'. That is, increasing the weight on B decreases the distortion on some subset B' of B while increasing the distortion elsewhere.

EXAMPLE

In this section we take up a two-signal numerical example that was used in Ref. 1 to illustrate multisignal MCESA. We use the same prior spectra and the same autocorrelation data, and we show plots of the posterior spectra that result from various choices of the weights. The autocorrelation data were derived from the sum of a pair of assumed original spectra S_S and S_B. (In Ref. 1 the indices stood for *signal* and *background*.) These spectra each have a sharp peak at a single frequency. They are shown in Fig. 1. The priors P_S and P_B were taken to be respectively flat and identical to S_B. They are shown in Fig. 2, which corresponds to Figs. 3 and 2 of Ref. 1. Figures 3–7 show poste[illegible] spectra Q_S and Q_B for five choices of constant weights: the ratios w_S/w_B are 90/10, 75/25, 50/[illegible]5/75, and 10/90. The spectra in Fig. 5, the equal-weight case, are identical to those in Figs. 6 and [illegible] Ref. 1. In each case there is a sharp peak in Q_B, corresponding to the peak that is in S_B and P_B [illegible]ach case there is a second peak in one or both of the posteriors, corresponding to the peak that [illegible] S_S but in neither prior; its appearance in the posteriors is due entirely to information contained i[illegible]tocorrelation data. In Fig. 3, for which the prior P_S is heavily weighted, the posterior Q_S is app[illegible]ely flat, resembling the prior, and most of the power in the second peak is attributed to Q_B. In F[illegible] P_B is the heavily weighted prior; most of the power in the second peak is attributed to Q_S, whose corresponding prior received a small weight indicating prior uncertainty. The figures between show a progression from the one extreme to the other as the weight shifts.

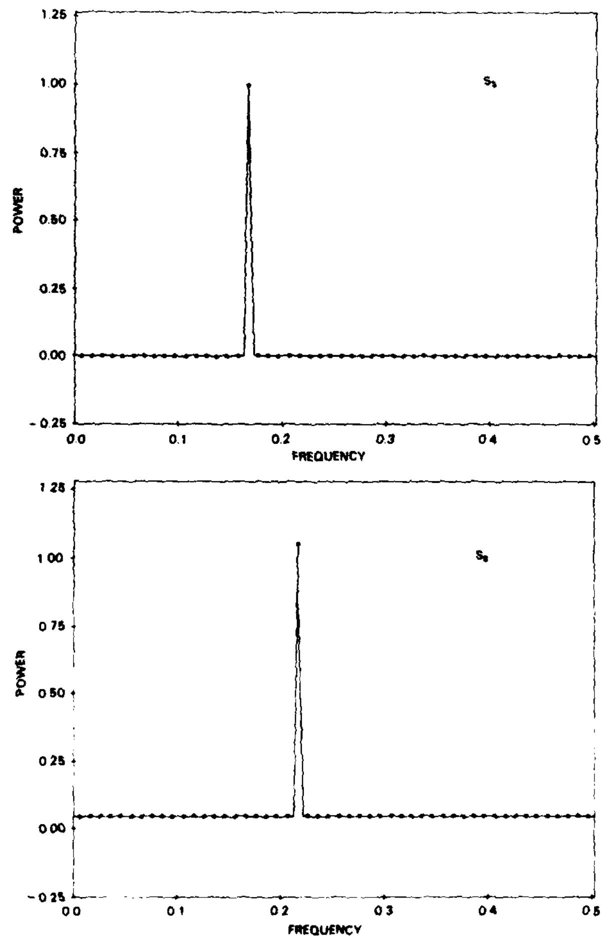

Fig. 1 — Assumed original spectra for example

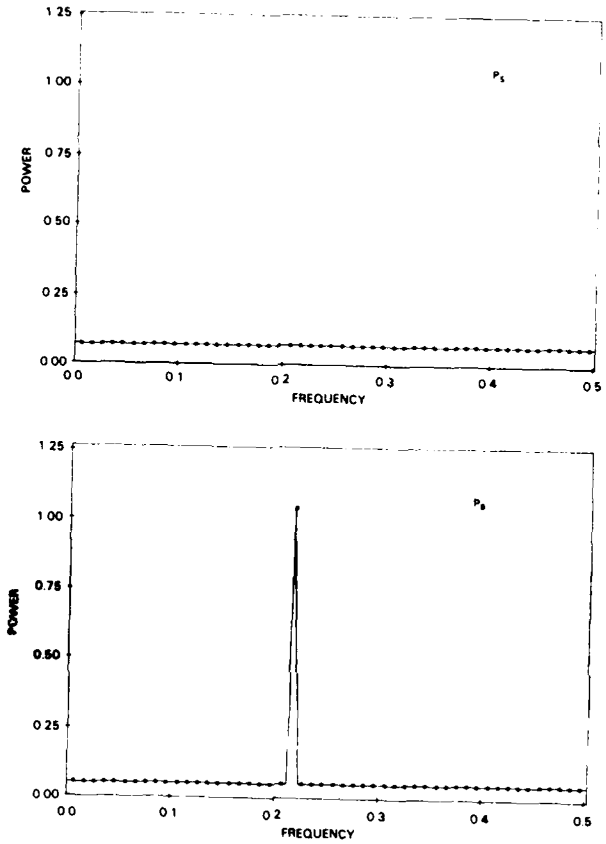

Fig. 2 — Prior spectral estimates

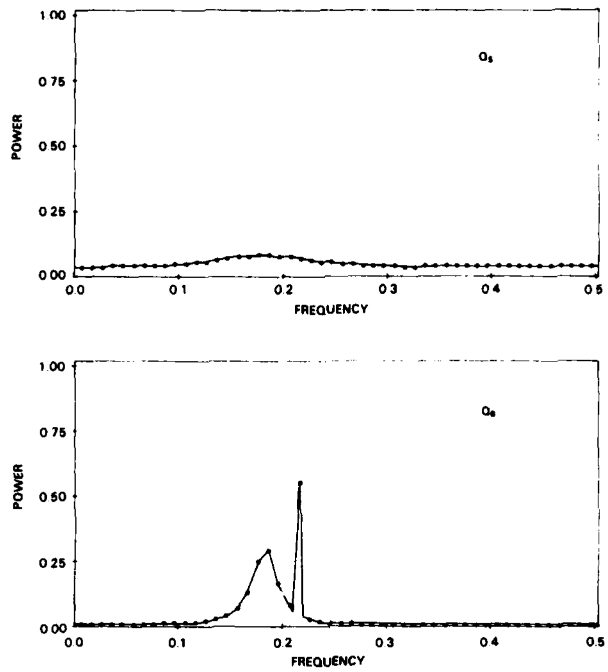

Fig. 3 — Posterior spectral estimates (w_S/w_B = 90/10)

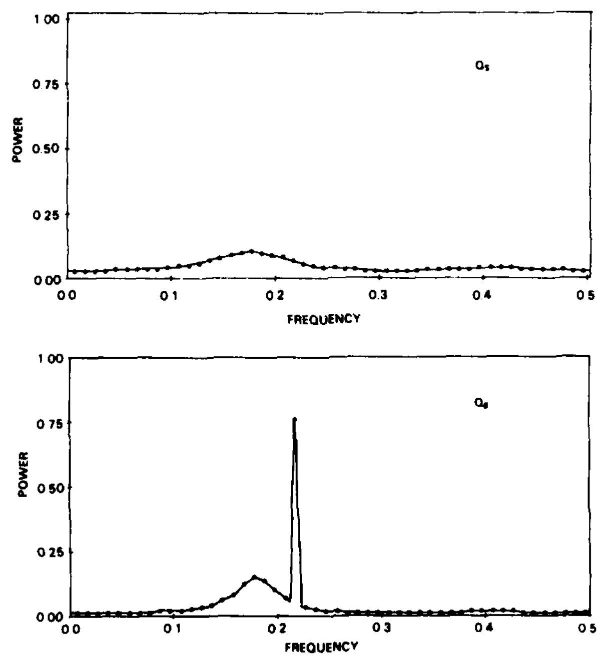

Fig. 4 — Posterior spectral estimates (w_S/w_B = 75/25)

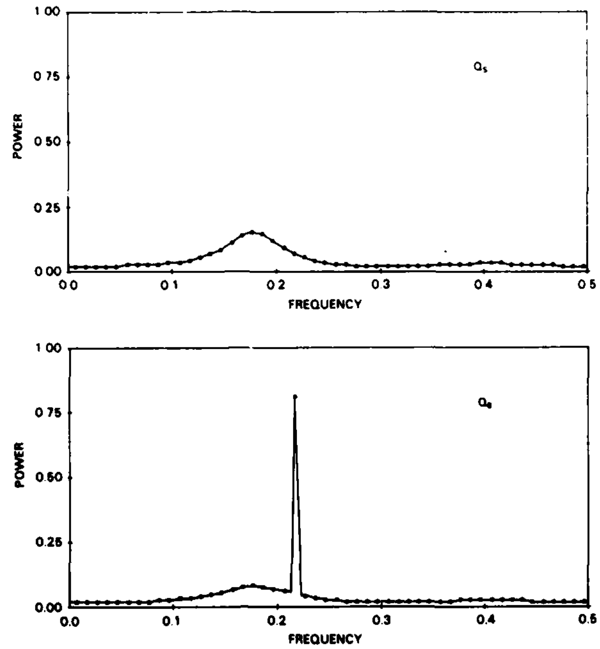

Fig. 5 — Posterior spectral estimates (w_S/w_B = 50/50)

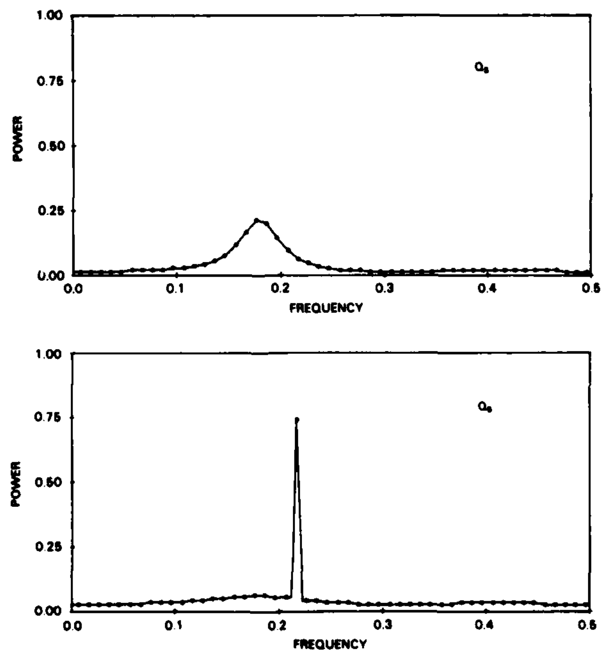

Fig. 6 — Posterior spectral estimates (w_S/w_B = 25/75)

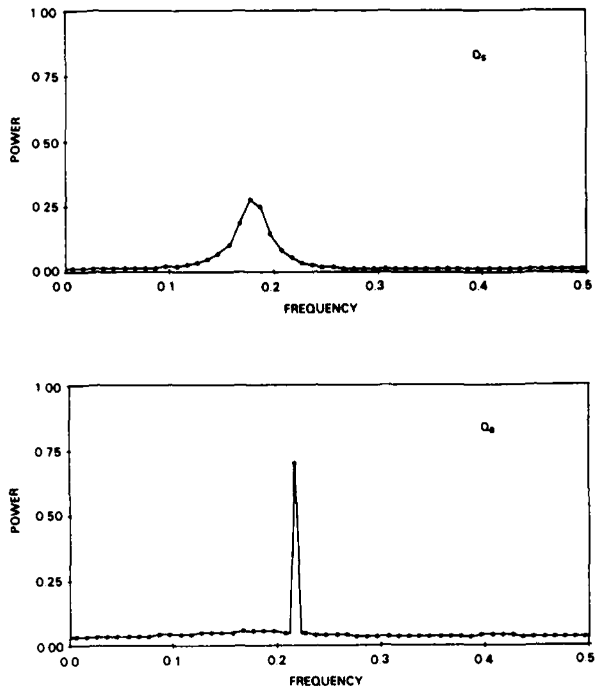

Fig. 7 — Posterior spectral estimates ($w_S/w_B = 10/90$)

DISCUSSION

The generalization of multisignal MCESA we have presented allows one to take into account the possibility that prior spectrum estimates are not equally reliable for all signals or in all frequency ranges for the same signal. One can express greater or less confidence or "degree of belief" in the prior estimates for the various signals in various frequency ranges by assigning greater or smaller values to the corresponding weights. Roughly speaking, increasing a weight tends to bring the corresponding posterior closer to the prior. This statement is made more precise in the results proved in the fourth section of this report and is illustrated in the examples.

Computer programs for multisignal MCESA spectrum estimation with weights may be found in the appendices of Ref. 7.

For a single signal, somewhat different considerations have led Chu and Messerschmitt [8] to the idea of minimizing a weighted Itakura-Saito distortion. However, their procedure involves minimizing the integral in Eq. (17) by varying P rather than Q. Chu and Messerschmitt view the frequency-dependent weight as a means for taking into account the varying perceptual importance of various frequency ranges of a speech signal. They consider the weighted distortion between a true spectrum and an all-pole estimate, rather than that between a posterior estimate and a prior estimate, and they thus obtain a new method for choosing all-pole estimates of the usual form, rather than an estimate in a new form such as Eq. (4).

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